$\angle OAG = \angle ODH$. $\therefore$ $GA = DH$. Similarly $CG = BH$.

$\therefore$ $AG + BH = CG + DH$.

$\therefore$ $\frac{1}{2}h(AG + BH) = \frac{1}{2}h(CG + DH)$.

$\therefore$ Area $AGHD$ = area $CGHD$.

Good solutions of this problem were received from *Professors Wm. Symmonds, and Cooper D. Schmitt.*

This problem has proved to be a very interesting one and for that reason we have given it extra space. The proposition to which Prof. Ellwood has given a proof explanatory to the proposition under consideration is known as the proposition of *Menelaus*. See *Halsted's Elementary Synthetic Geometry*, p. 117. Editor.

PROBLEMS.

42. Proposed by G. I. HOPKINS, Instructor in Mathematics and Physics in High School, Manchester, New Hampshire.

If the bisectors of two angles of a triangle are equal the triangle is isosceles.

[The term *bisector* in this theorem means the line which divides an angle into two equal parts and terminates in the opposite side.]

43. Proposed by J. F. W. SCHEFFER, Hagerstown, Maryland.

The consecutive sides of a quadrilateral are a, b, c, d. Supposing its diagonals to be equal, find them and also the area of the quadrilateral.

CALCULUS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

27. Proposed by Professor G. B. M. ZERR, A. M., Principal of High School. Staunton, Virginia.

A runs around the circumference of a circular field with velocity m feet: B starts from the centre with velocity $n > m$ feet to catch A. The straight line joining their positions always passes through the centre. Find the equation to the curve described by B, the distance he runs and the time occupied.

I. Solution by A. H. HOLMES, Brunswick, Maine, H. W. DRAUGHON, Olio, Mississippi, and the PROPOSER.

Let A be the point of starting of the pursued, P, B, the position of the pursuer and pursued at any time.

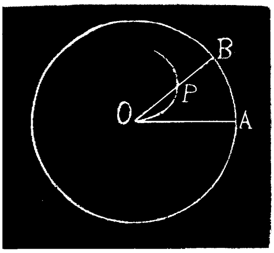
Let $OA = a$, $OP = r$, $\angle BOA = \theta$, $\frac{m}{n} = u$, arc $OP = s$.

Then $us=a\theta\ldots(1)$. $\therefore$ $s=\dfrac{a\theta}{u}=\dfrac{an\theta}{m}$ is the intrinsic equation to the curve.

From (1) $\dfrac{ds}{d\theta}=\dfrac{a}{u}=\sqrt{r^2+\left(\dfrac{dr}{d\theta}\right)^2}$.

$$\therefore\quad r^2+\left(\frac{dr}{d\theta}\right)^2=\frac{a^2}{u^2}\text{ and }d\theta=\frac{dr}{\sqrt{\left(\dfrac{a^2}{u^2}-r^2\right)}}.$$

$\therefore$ $\theta=\sin^{-1}\dfrac{ur}{a}\ldots(2)$.



$\therefore$ $r=\dfrac{a}{u}\sin\theta=\dfrac{an}{m}\sin\theta$, is the polar equation and $m(x^2+y^2)=any$, is the rectangular equation.

The value of θ from (2) in (1) gives

$$s=\frac{a}{u}\sin^{-1}\frac{ur}{a},$$

the length for any value of r. When $r=a$,

$$s=\frac{a}{u}\sin^{-1}u=\frac{an}{m}\sin^{-1}\frac{m}{n}=\text{distance run.}$$

$$t=\text{time}=\frac{s}{n}=\frac{a}{m}\sin^{-1}\frac{m}{n}.$$

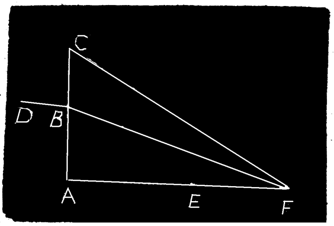
Also solved by *Professors O. W. Anthony, J. Scheffer, and William Symmonds*.

28. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

How far from the stage must Miss Love sit in order that she *may see to best advantage* Mr. Rich deliver the valedictory oration?

Solution by O. W. ANTHONY, Missouri Military Academy, Mexico, Missouri, and the PROPOSER.

Let E represent the position of Miss Love's eyes; DB the stage from which Mr. Rich orates; $AB,=m$ feet, the height of the stage above Miss Love's eyes; $BC,=n$ feet, the height of Mr. Rich; and $AE,=x$ feet, the required distance In order that Miss Love may *see* Mr. Rich *to best advantage*, the angle BEC must be a maximum, that is,



$$U=\tan^{-1}\left(\frac{m+n}{x}\right)-\tan^{-1}\left(\frac{m}{x}\right)=\text{a Maximum.}$$

$$\therefore\quad\frac{dU}{dx}=\frac{m}{x^2+m^2}-\frac{m+n}{x^2+(m+n)^2}=0\ldots(1).$$

Whence $x=\sqrt{[m(m+n)]}$ feet, which is the required distance.

29. Proposed by CHARLES E. MYERS, Canton, Ohio

A hen running at the rate of $n=2$ feet per second, on the circumference of a circle, radius $r=50$ feet, is observed by a hawk $a=600$ feet directly above the center.